Modeling the Effects of Cathode Wall Nonuniformities in Magnetohydrodynamic Generators

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Abstract

THE effect of cathode wall nonuniformities on the electrical performance of Faraday loaded generators is modeled analytically. An approximate three-dimensional model is formulated to determine the distributions of the electrical variables within the magnetohydrodynamic (MHD) channel in the presence of cathode nonuniformities. Nonuniformities in this model are treated as a coarser resegmentation of the cathode wall. In addition, a two-dimensional electrical model is used to study the effects of cathode slag shorting on local electrical characteristics. These two models are described and calculated results using both models are compared with experimental data.

Contents

Cathode nonuniformities appear in MHD generators operating with slag-laden flows. These nonuniformities originate at the cathode wall when groups of electrodes are shorted by electrically polarized slag coatings.[1] The resulting change in the effective segmentation of the cathode wall causes a few insulator gaps to sustain the total Hall voltage of the generator. In this paper, an approximate three-dimensional electrical model of cathode nonuniformities is formulated. This electrical model has been incorporated into an MHD design and prediction computer code[2] in order to assess the effects of these nonuniformities on the overall Faraday generator performance.

The effects of cathode nonuniformities can be modeled by arbitrarily increasing the slag-layer electrical conductivity to account for slag polarization. However, the approach of this paper is to model the slag polarization-cathode nonuniformity effects as a change in the effective segmentation of the cathode wall. The approximate three-dimensional model is a finite-region integral model formulated to study the effects of finite segmentation. In order to analyze finite segmentation effects via this finite element model, transverse profiles of the electric fields and currents are required.[3] A two-dimensional electrodynamic model was used to perform a parametric study of these electrical profiles. Four parameters were investigated: the number of shorted cathodes, Hall parameter, size of the generator, and wall electrical conductivity. The objective of this two-dimensional calculation is to identify the transverse variation of the Hall electric field (E_x) distribution over the electrodes and the Faraday current density (J_y) distribution over th einterelectrode insulators. In addition, electrical boundary-layer lengths of these distributions are sought for use as a profile cutoff in the finite-region model. These electrical boundary-layer lengths (ℓ_E^* and ℓ_J^*) are defined as the distances over which the Hall field or Faraday current density build up to the core values. In what follows each of the two models will be described briefly, and comparisons of the results of the combined model and experimental results will be presented.

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Finite-Region Electrical Model of Cathode Wall Nonuniformities

An integral technique has been formulated to determine the distributions of the electrical variables within the MHD channel with a slag shorted cathode wall. These solutions are obtained from a closed set of equations that resulted from the description of the Faraday load connection and the integration (averaging) of model electrical distributions across various finite regions. The finite element regions are formed by subdividing the calculational domain into various boundary layers, slag layers, and core flow regions. The distributions of the electrical variables in these regions are determined such that Ohm's law and certain additional electrical boundary conditions are satisfied.

In the electrode wall regions, a blending function approach is used to prescribe the transverse variations of Hall field and Faraday current density. The lengths ℓ_{EA}^* and ℓ_{EC}^*, ℓ_{JA}^*, and ℓ_{JC}^* (the electrical boundary-layer lengths previously mentioned, where A and C stand for Anode and Cathode, and c stands for the core region) determine the dimensions of the finite regions. For example, above the cathode insulators,

$$J_{y_{IC}}(x,y) = J_{y_c}(x)\phi_{JC}(y) \tag{1}$$

and above the conductors,

$$E_{x_{EC}}(x,y) = E_{x_c}(x)\phi_{EC}(y) \tag{2}$$

The ϕ's are matching functions that go to zero on the wall and to one at the edge of the electrical boundary layer. From electric field irrotationality and current conservation arguments, $E_{x_{IC}}$ and $J_{y_{EC}}$ can be approximated as:

$$E_{x_{IC}}(x,y) \cong E_{x_c}(x)\left\{1+\frac{N(\ell_i+\ell_e)-\ell_i}{\ell_i}[1-\phi_{EC}(y)]\right\} \tag{3}$$

$$J_{y_{EC}}(x,y) \cong J_{y_c}(x)\left\{1+\frac{\ell_i}{N(\ell_i+\ell_e)-\ell_i}[1-\phi_{JC}(y)]\right\} \tag{4}$$

where ℓ_e and ℓ_i refer to the conductor and insulator axial dimensions, and N is the number of cathode conductor segments shorted by polarized slag. The remaining current densities and electric fields in the electrode wall regions are determined from the preceding model distributions and Ohm's law, given the velocity, conductivity, and Hall parameter distributions through the boundary layers. Proper values for the matching functions ϕ and the ℓ^*'s as functions of β, pitch-to-height ratio, number of shorted cathodes N, etc., are established from the two-dimensional electrical model to be described subsequently.

The sidewalls of the MHD generator are assumed to be electrically insulating so the core electric field can be imposed on the sidewall boundary-layer regions. For the corner flow regions, assumptions are made to preserve current conservation and to be consistent with electrode wall and sidewall boundary-layer fields. The electrical variables in the slag layers are determined from electrical conditions imposed from the boundary-layer regions immediately above them. The expressions for the electric fields and current densities can be integrated (averaged) in their respective regions, given a suitable form of the blending functions ϕ.

This finite-region electrical model has been used to investigate the effects of cathode slag shorts on the local electrical characteristics in an Avco Mk VI subsonic generator. The sensitivity of the electrical solution to the number of assumed electrode shorts on the cathode wall was investigated and the results are shown in Fig. 1. These results are obtained for the midchannel conditions of the generator. The sidewall and anode slag layer conductivities were taken to be 10 mho/m. The estimated values for the characteristic lengths, ℓ^*s, and for the profiles of the blending functions, ϕ's, were obtained from the results of the two-dimensional current stream function solver explained in the following subsection. It is observed that as the number of slag shorts increases, the flow of axial current in the generator increases. The corresponding decrease in the magnitudes of the other electrical variables as the axial currents increase is also shown in Fig. 1.

Two-Dimensional Electrical Model of Cathode Wall Nonuniformities

A two-dimensional electrodynamic model was formulated to obtain the internal electrical characteristics within the midchannel plane perpendicular to the magnetic field (xy plane). The model seeks the electrical solutions for Faraday loaded generators with slag shorted cathodes given the gasdynamic distributions of velocity, Hall parameter, and conductivity. Maxwell's equations and Ohm's law are combined to get the following partial differential equation:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{1}{\sigma}\frac{d\sigma}{dy}\frac{\partial \psi}{\partial y} - \sigma \frac{d(\beta/\sigma)}{dy}\frac{\partial \psi}{\partial x} = 0 \qquad (5)$$

where ψ is the current stream function, β the Hall parameter, and σ the electrical conductivity. In Eq. (5), only transverse (y) variations in conductivity, Hall parameter, and gas velocity are assumed because the axial variations in σ, β, and u are expected to be small.

A finite difference method is used to solve the partial differential equation, which includes variable grid sizes in both the x and y directions. The set of simultaneous linear equations and unknowns which evolves is solved by directly inverting the resulting coefficient matrix using a specially designed sparse matrix algorithm. Appropriate boundary conditions are used: $E_x = 0$ over anodes and shorted cathode and $J_y = 0$ over insulators. Inputs to the model are the core Hall parameter, Faraday load currents, geometric nondimensionalized lengths (p/H, insulator length/p, boundary-layer thickness/H), and the conductivity and velocity profiles (H is the channel height, p the electrode pitch).

Results show that for the Hall electric field, ℓ_E^* varies slowly with the number of cathode shorts and is approximately equal to the pitch length. For the current density, ℓ_J^* increases linearly with N and is thus proportional to the effective cathode pitch $\sim Np$. The effect of MHD generator size was studied by varying the pitch-to-height ratio (p/H). Results show that all of the nondimensional electrical lengths diminish as the generator is scaled up in size. This is caused by the fact that the electrical boundary layers scale with the p/H ratio rather than with the thermal or velocity boundary-layer thickness.

Combined Model Results

The results of the generator performance prediction using the new electric model (composed of both the finite element calculation and the curve fits for the two-dimensional solution of the characteristic lengths and profiles) are shown in Table 1 for a Mk VI supersonic channel and the Component Development and Integration Facility CD/F 1A1 channel.[4] These results use the experimentally measured distribution of N (N = number of electrodes within a shorted cathode group) as input. Relatively good agreements in both power output and Hall voltage are obtained between analysis and data. This agreement was achieved without resorting to large wall slag conductivities which are needed with infinite segmentation electrical models.

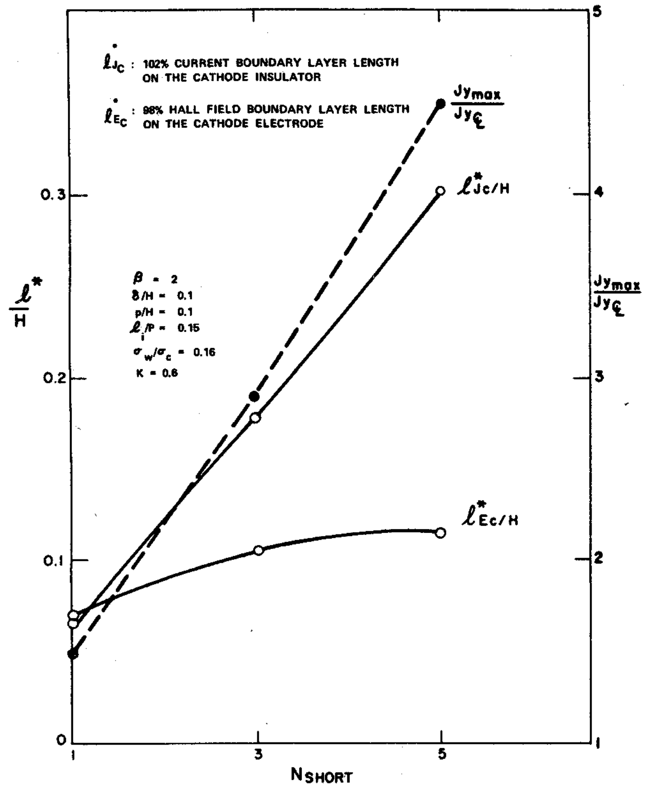

Fig. 1 Dependence of core electrical properties with number of shorted cathodes.

Table 1 Performance comparison between analysis and experiment

	MkVI[a]					
	Data	Calculation				
Electric power, kW	315	304				
Hall voltage, kV	1.78	1.75				
Channel heat loss, MW	2.15	1.93				
	CDIF 1A1					
	$N/O = 0.78$		$N/O = 0.88$		$N/O = 0.97$	
	Data	Calculation	Data	Calculation	Data	Calculation
Power, MW	2.12	2.12	1.80	1.68	1.52	1.39
V hall, kV	6.31	6.05	7.35	6.79	8.02	7.23
Q loss, MW	6.20	6.25	6.02	5.87	5.66	5.54

[a]Mass flow rate = 2.86 kg/s, $N/O = 0.6$, $B_{max} = 2.9$ T.

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